Self-equalizing headways reduce bus-bunching

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Abstract

The primary challenge for an urban bus system is to maintain constant headways between successive buses. Most bus systems try to achieve this by adherence to a schedule; but this is undermined by the tendency of headways to collapse, so that buses travel in bunches. To counter this, we propose a new method of coördinating buses. Our method abandons the idea of a schedule and even any a priori target headway, and instead focuses on equalizing headways. Our method is self-managing: It allows the natural headway of the system to emerge spontaneously; and after disturbances it reëqualizes headways without intervention by management or even awareness of the drivers.

We report on a successful implementation to control a bus route in Atlanta.

Keywords: Bus bunching, transit operations, adaptive control, self-organization
1 Bus bunching

When buses circulate on a route, service is best when times between successive bus arrivals (headways) are equal. However, it is impossible to maintain equal headways because of unavoidable variability in traffic and in the boarding and deboarding of passengers. These variations inevitably cause some buses to slow relative to others; and the larger the headway, the more strongly it tends to grow, because the trailing bus will likely meet more passengers than average at each stop and so will be further delayed. Similarly, a headway that has become smaller will tend to shrink even more because the trailing bus will likely meet fewer passengers. This phenomenon is called bus bunching or platooning.

Bus bunching increases both the mean and variance of the wait time of passengers. This is because larger gaps attract proportionally more passengers, and arrivals within these larger gaps will on average have to wait longer, and will experience less certainty about the next bus arrival. Bus bunching also wastes capacity, as the lead bus of a bunch can be full, while the trailing buses are nearly empty. At the time of this writing, bus bunching is the most common customer complaint at the Chicago Transit Authority (CTA), and has received considerable local press coverage (see for example Gerasole (2008) and Luman (2007)).

We propose a method of coördinating buses that tends to equalize headways and so provides better service. Our system is unusual in that it abandons the concept of a schedule. This makes sense where the planned headway on routes is small, say 5–10 minutes, in which case riders typically ignore any schedule (Minser (2009)). By focusing on equalizing the headways, buses visit each stop at more regular intervals, and therefore no passenger has to wait too long.

In fact, under our scheme the headways are “self-equalizing”: After any disruption the buses will spontaneously re-space themselves to equalize headways, and will do so without direction from management or the intention or even awareness of the drivers. Thus management is freed from building and monitoring a schedule and drivers are freed
from the distraction of repeatedly checking their time, location, and velocity.

2 Coördinating buses

Bus bunching has long been understood (for example, Newell and Potts (1964)); but it is hard to fix. There are essentially two ways to separate buses that are too close. One is to ask the leading bus to speed up; but this can be disruptive to traffic flow, and certainly difficult or impossible in a heavily-trafficked urban environment.

The other way to separate buses that are too close is to slow the trailing bus. But a slowing bus can annoy both following traffic and on-board passengers, so in practice buses are typically delayed, when necessary, only at certain key stops, which we refer to as control points. (The endpoints of routes that follow an out-and-back path are ideal for delaying a bus because few passengers ride through them.)

Previously, there have been two approaches to calculating the delay incurred by a bus arriving at a control point: either attempt to adhere to a target schedule or a target headway. In the US, municipal bus routes are typically managed by target schedules, in which the arrival times of buses at each stop are planned to the minute and extra time is budgeted for each bus to pause at each control point. These delays help recover the schedule if the buses get ahead; and if behind, a bus can forfeit some or all of the planned delay. More planned delay gives greater ability to recover a schedule, but at a cost of idle bus capacity.

Recently, Daganzo (2009) proposed an approach that abandons the concept of a schedule to focus on achieving a target headway that has been specified in advance. When a bus arrives at a control point, it compares its headway to a pre-specified target value. If the headway is smaller, then the arriving bus is judged to be following too close behind its predecessor, and it delays for a longer than nominal duration; and if the headway is larger, the arriving bus delays a shorter than nominal value.

Both target schedules and target headways attempt to realize a pre-specified static
value of headway. We consider this a technical weakness in both approaches, because
the ideal achievable headway is not static and not even knowable in advance. Instead, it
changes continually with traffic conditions, habits of the driver, and numbers of passengers
boarding and deboarding at each stop. Consequently, any system that coördinates buses
based on target headways must sometimes underestimate achievable headway, and so fail
to meet the target, and sometimes overestimate it, and so waste bus capacity.

A more serious objection is that neither a target schedule nor target headway can
respond adequately to serious disruption. For example, when a bus breaks down, it leaves
a hole in the schedule until a replacement can be inserted. When the gap is large enough,
it will overwhelm any planned slack. In the case of a target schedule, a trailing bus may
be so far behind schedule that its planned departure time will have already lapsed when it
reaches a control point, and it will depart immediately, still behind schedule. And in the
case of a target headway, the delay computed for the trailing bus can be negative, which
can be interpreted as directing the trailing bus to speed up — but this is not practical
except in special situations, such as when there are reserved bus lanes. This leaves both
schemes vulnerable to any large system-wide disruption, such as a snowstorm, that might
reduce average bus velocity, and so increase all headways. When disruptions are large,
both target-based schemes abdicate control, and the result is bus bunching.

We propose a control system that abandons both the notion of a schedule and, in
addition, that of any pre-specified target headway. By abandoning any target headway, the
system is free to express the natural headway, which may vary from moment to moment.
Moreover, under our scheme headways will tend to equilibrate even in the presence of
perturbations. Our system will converge to the smallest common headway possible given
the current capacity and demands upon the system. Even after large shocks, such as if
a bus were to break down, our system will spontaneously re-position buses to achieve a
new, albeit necessarily larger, common headway.

In Section 3 we explain the basic mechanism of our scheme and analyze how it works
in an idealized model in which perturbations are infrequent (Section 3.1). Then we enhance this model to explore the contending tendencies of equalization and a very strong form of bunching (Section 3.2). Finally, we strengthen our scheme to produce the version that we suggest for practical use (Section 3.3).

Of course the true test of such a scheme is how well it works. In Section 4 we report on the performance of self-equalizing headways on the central bus route through the campus of the Georgia Institute of Technology in Atlanta, Georgia, US. In Section 5 we compare the three approaches—target schedule, target headway, and self-equalizing headways—by simulating ridership and traffic on route 63 of the Chicago Transit Authority.

3 Self-equalizing headways

As do the target schedule or target headway approaches, our method seeks to improve service by systematically delaying buses at control points.

Consider a route with a single control point. Let the bus newly-arrived at the control point be bus 1, and index the others in the direction of travel, so that the bus trailing 1 is bus \( n \). The headway \( h_i \) is the time separating bus \( i \) from bus \( i + 1 \). We base the delay of bus 1 on the headway \( h_n \) of the trailing bus (or, in the terminology of some authors, on the backwards headway of bus 1). Specifically, we delay bus 1 at the control point for duration

\[
\alpha h_n, \tag{3.1}
\]

where \( 0 < \alpha < 1 \) is a control parameter that determines the sensitivity of our scheme to perturbations.

To understand the behavior of buses under our scheme, we model their movement as a dynamical system. Consider each of \( n \) buses moving at constant velocity \( \bar{v} \) about a circular route of length that has been normalized to 1, with a single control point at 0 (equivalently, 1). At any point in time each bus \( i \) has a location \( x_i \in [0, 1) \) about this circuit.
We restrict attention to those points in time (or iterates) when a bus arrives at the control point 0. At each such time we re-index the buses so that the bus that has just arrived at the control point is bus 1, the next in the direction of bus movement is bus 2, and so on, until the last bus, which is the next bus to arrive at the control point, is bus n.

Let those instants at which a bus arrives at the control point be indexed by \( t = 1, 2, \ldots \). For each time \( t \) let the vector \( \mathbf{x}' = (x'_1, x'_2, \ldots, x'_n) \) represent the locations of the buses, where \( 0 = x'_1 \leq x'_2 \leq \cdots \leq x'_n < 1 \).

From arbitrary starting positions \( \mathbf{x}^0 \), the trajectory of bus positions \( \{\mathbf{x}^0, \mathbf{x}^1, \ldots\} \) may be thought of as a series of snapshots of the bus route at those times when a bus arrives at the control point. Let \( f \) be the implicit function that maps the bus positions from one iterate to the next, so that \( \mathbf{x}^{t+1} = f(\mathbf{x}') \).

Let the vector \( \mathbf{h}' = (h'_1, h'_2, \ldots, h'_n) \) give the headways of the buses at time \( t \). In the absence of perturbations, \( h'_i = (x'_{i+1} - x'_i) / \bar{v} \) for all buses \( i \), except perhaps for bus 1, which we require to pause at the control point for time \( \alpha h'_n \) according to (3.1).

### 3.1 Equilibrium dynamics of self-equalizing headways

In the absence of perturbations we can write the dynamics equations using our computed delay (3.1) as

\[
\begin{align*}
\mathbf{x}^{t+1} &= 0 \\
\mathbf{x}^{t+1}_2 &= x'_1 + (h'_1 - \alpha h'_n) \bar{v} \\
&= h'_n (1 - \alpha) \bar{v} \\
\mathbf{x}^{t+1}_i &= x'_{i-1} + h'_i \bar{v} \quad \text{for each } i = 3, \ldots, n.
\end{align*}
\]  

(3.2)

Our main result says that from any initial positions of the buses, a common headway will spontaneously emerge, and the value of this headway depends only on the number and average velocity \( \bar{v} \) of the buses and on the control parameter \( \alpha \).
Theorem 3.1. For \(0 < \alpha < 1\), any trajectory of bus positions will converge to a unique fixed point \(x^*\) with common headway

\[
h^* = \frac{1}{(n - \alpha)\bar{v}}.
\]  

(3.3)

Proof. The successive headways can be determined by dividing the distance traveled by each bus by its velocity \(\bar{v}\) (except for the first bus, which has effective velocity \((1 - \alpha)\bar{v}\) because of the delay at the control point). Using Equations (3.2) we can write

\[
h_t^{i+1} = \frac{x_t^{i+1} - x_1^{i+1}}{(1 - \alpha)\bar{v}} = \frac{h_n'(1 - \alpha)\bar{v} - 0}{(1 - \alpha)\bar{v}}
\]

\[= h_n'
\]

\[
h_2^{i+1} = \frac{x_3^{i+1} - x_2^{i+1}}{\bar{v}} = \frac{x_2^{i} - x_1^{i}}{\bar{v}} + h_n' - (1 - \alpha)h_n'
\]

\[= \frac{x_2^{i} - x_1^{i}}{(1 - \alpha)\bar{v}} (1 - \alpha) + ah_n'
\]

\[= (1 - \alpha)h_1' + ah_n'
\]

\[
h_3^{i+1} = \frac{x_4^{i+1} - x_3^{i+1}}{\bar{v}} = \frac{x_3^{i} - x_2^{i}}{\bar{v}} + \frac{h_n'\bar{v}}{\bar{v}} - \frac{h_n'\bar{v}}{\bar{v}}
\]

\[= h_2'
\]

\[
h_i^{i+1} = h_{i-1}' \quad \text{for each } i = 4, \ldots, n.
\]

These equations can be written as a linear system \(h^{i+1} = A h'\), where \(A\) is a stochastic matrix that may be interpreted to represent the transitions of a finite state Markov chain that is irreducible and aperiodic. By the Markov Chain Theorem each row of the limit \(A^\infty\) is

\[
\begin{pmatrix}
1 - \alpha \\
n - \alpha \\
n - \alpha \\
n - \alpha
\end{pmatrix}
\]

and all entries of \(h'\) converge to the claimed common value. \(\square\)

The smallest possible common headway for \(n\) buses, each traveling at velocity \(\bar{v}\) is \(1/(n\bar{v})\). But such a system has no slack and so no ability to recover from disruptions. Under our scheme headways converge to the common value \(h^*\) as given in Expression (3.3).
From the denominator it may be seen that $\alpha$ represents the bus capacity held in reserve at the control point to help the system recover from disruptions.

Note that the equilibrium value $h^*$ is determined by several factors. Slack, as given by $\alpha$, is set by management. On the other hand, the value of $\bar{v}$ is mostly determined by traffic conditions and levels of ridership. And while management can choose the number $n$ of buses on the route, unplanned events, such as bus breakdowns, may have an effect as well. But however the terms of $h^*$ may be determined, under our scheme all headways will be pulled towards this common value.

### 3.2 Super-linear bus bunching

In section 3.1 we established the tendency under our scheme of headways to equalize. Here we show that our scheme can resist even a very strong form of bunching.

In a study of route 63 for the CTA, Milkovits (2008) measured the time a bus spends boarding and deboarding passengers at a stop. He found that passenger movement was hindered by those on the bus, and so dwell time at the stop was not simply proportional to the number of boarding/deboarding passengers, as is typically assumed. Instead, he found that dwell time varied as the square of the number of passengers riding on the bus multiplied by the total number boarding and deboarding. Therefore, if the number of passengers boarding or deboarding grows linearly with the time since the last bus, the headway must grow super-linearly.

To model this phenomenon, let the effective average velocity of bus $i$ during iteration $t$ be

$$\bar{v} \left( \frac{x_{i+1} - x_i}{\bar{v}} \right)^{\omega}$$

for some $\omega \geq 0$. The expression $(x_{i+1} - x_i)/\bar{v}$ measures the nominal linear growth in the headway, and $\omega$ expresses a super-linear decline in the velocity of the bus as the number of passengers increase. This means the velocity of a bus increases rapidly if its headway decreases.
Thus there are two opposing forces: The dynamics of system (3.2) pull the buses toward a state of equal headways when $0 < \alpha < 1$ and the super-linear dwell times pull the buses towards bunching when $0 < \omega$. This contention is described by the following system of equations.

\[
x_{1}^{t+1} = 0 \\
x_{2}^{t+1} = \min \left\{ x_{1}^{t} + \frac{(T^{t} - \alpha T^{t}) \bar{v}}{((x_{2}^{t} - x_{1}^{t})/\bar{v})^{\omega}}, x_{3}^{t+1} \right\} = \min \left\{ x_{1}^{t} + \frac{T^{t} (1 - \alpha \bar{v})}{((x_{2}^{t} - x_{1}^{t})/\bar{v})^{\omega}}, x_{3}^{t+1} \right\} \\
x_{i}^{t+1} = \min \left\{ x_{i-1}^{t} + \frac{T^{t} \bar{v}}{((x_{i}^{t} - x_{i-1}^{t})/\bar{v})^{\omega}}, x_{i+1}^{t+1} \right\} \quad \text{for each} \quad i = 3, \ldots, n - 1 \\
x_{n}^{t+1} = \min \left\{ x_{n-1}^{t} + \frac{T^{t} \bar{v}}{((x_{n}^{t} - x_{n-1}^{t})/\bar{v})^{\omega}}, 1 \right\} ,
\]

where

\[
T^{t} = \frac{(1 - x_{n}^{t})}{\bar{v}/((1 - x_{n}^{t})/\bar{v})^{\omega}} = \frac{(1 - x_{n}^{t})^{1+\omega}}{\bar{v}^{1+\omega}}
\]

is the time required for iteration $t$, and the min operator prohibits any bus from passing another. (This better mimics actual behavior of buses because, if passing is allowed, the faster bus would slow down as soon as it assumed the lead because it would face a larger headway, and so more waiting passengers.)

The dynamics equations (3.6) are complex, but can be examined numerically to observe how the opposing forces of equalization ($\alpha$) and bunching ($\omega$) interact. Figure 1 is typical in showing a clear boundary separating the regions of bunching and equalization. It represents the limiting behavior of the dynamics equations (3.6) for different starting positions of four buses (the horizontal axis) versus different strengths $\omega$ of bunching (the vertical axis). The x-axis gives the starting position $x_{0}^{2}$ of the second bus. The third and fourth are equally spaced, so that the buses start in positions $(0, x_{2}^{0}, 2x_{2}^{0}, 3x_{2}^{0})$. For values of $x_{2}^{0}$ close to 0.25, the buses begin with headways that are nearly equal, and the system tends to stay balanced. But even if buses begin more bunched they will converge to equalized headways if the tendency $\omega$ to bunch is not too large.
Figure 1: There is a clear boundary separating eventual bunching from self-equalization. When the buses start with nearly equal headways (right side of the graph), then equalization overpowers the tendency to bunch and the system converges to equal headways. When initial headways are more out of balance, bunching can overpower our scheme (left side of the graph).
3.3 Strengthening headway equalization

While our computation of delay as $\alpha h_n$ resists bunching, its effect can be strengthened by a simple enhancement. Equal headways will be restored more quickly after large disruptions if we add the stipulation that successive buses departing the control point be separated by at least $\beta > 0$ time units, in which case our scheme works as follows:

Whenever a bus arrives at a control point it must wait

- for duration $\alpha h_n$; or
- until $\beta$ time units after the previous bus has departed the control point,

whichever is greater.

It is advisable to set $\beta$ smaller than any value of headway expected to emerge naturally. For example, it might be set to the headway expected if there were no other traffic and no passengers. So long as $\beta$ has been set to a value no greater than $h^*$, the headways will still self-equalize.

**Corollary 3.2.** In the absence of further perturbations headways will equalize under the extended scheme as long as $\beta < h^*$.

**Proof.** After $n$ iterations all headways will be larger than $\beta$, and will remain so because, under our method, in the absence of further perturbations, the minimum headway never decreases. All subsequent dynamics are as described in the proof to Theorem [3.1] □

The effect of the $\beta$ control is to restrict the system of buses to the righthand region of Figure[1] which favors evolution of equalized headways.
4 Performance on a public bus route

We tested our coordination scheme on the central bus route at the Georgia Institute of Technology. This route is an out-and-back loop of total length 3.3 miles (5.3 kilometers). It cuts through the center of campus and ties together key origins and destinations, including dormitories, the Student Athletic Center, Technology Square, and the Atlanta subway. This is the most heavily traveled of the campus bus routes with around 5,000 riders each day. In addition to the usual morning and evening peaks, the route experiences surges in ridership ten minutes before and ten minutes after class changes.

To help the buses keep to schedule, the operator of the bus system (Groome Transportation) maintains two control terminals, one at each of the endpoints of the route, ISyE/Rec Center and MARTA Midtown, where the buses typically pause for a few minutes. Furthermore, a manager is stationed at the ISyE/Rec Center stop to monitor performance and deal with problems.

This route is normally run according to a schedule. Each driver is assigned a bus and each bus is assigned a repeating sequence of bus stops and corresponding times. The key performance indicator for the system and for the individual drivers was adherence to schedule.

For our experiment, we instructed the drivers to abandon the schedule and ignore headways. Instead, they were to simply drive with the flow of traffic from one end of the route to the other. A student at each control point recorded arrival times and computed departure times. This computation relies on having estimates of how long until the next bus arrives to a control point. We got this information from www.NextBus.com, which collects the positions of buses from their GPS devices every 15 seconds, predicts the number of minutes until the next bus will arrive, and updates a publicly-accessible web page.
Figure 2: Standard deviations of time between bus departures along each of the 19 stops along an out-and-back route, where Stop #1 is the westernmost endpoint and stop #10 the easternmost. Under our scheme (the lowest line, in red), both the average headways and the standard deviations were reduced compared to the same day of the week in the two preceding weeks. In other words, service was both better and more reliable.

4.1 Experiment 1: Regularity of service

To be conservative we configured our scheme to approximate the normal schedule. During most of the day during the academic year there are $n = 6$ buses circulating on this route. The target schedule calls for 6 minute headways and a 3 minute wait at one of the control points and a 4 minute wait at the other. We approximated this behavior by setting $\alpha = 0.5$ and $\beta = 5$ minutes at each of the two control points.

As shown in Figure 2, buses departed each control point with greater regularity under our scheme than under the target schedule. Furthermore, our headways were on average shorter than headways realized under the target schedule. Thus passenger waiting times were shorter and more reliable.

Our scheme entirely avoided severe bunching: The smallest headway under our scheme
was 2 minutes 28 seconds, compared to only 1 second under scheduled service on a comparable day. Similarly, the largest headway under our scheme was 14 minutes 59 seconds, but 19 minutes 45 seconds under scheduled service.

We interviewed bus drivers after the experiment and they liked not having to worry constantly about the schedule, checking the clock, and making adjustments. They felt that our scheme freed them to concentrate on safe driving.

4.2 Experiment 2: Resilience

In both the Georgia Tech and the CTA systems, buses continually circle an assigned route. If a bus is suddenly and unexpectedly unavailable, such as due to mechanical breakdown, a portion of the schedule remains unserved, so that an observer at a bus stop sees a recurring gap in service. Under our scheme, the remaining buses spontaneously reposition themselves to equalize headways without any direction by management or intention of the drivers. Indeed we observed this behavior in a subsequent experiment with the Georgia Tech Bus Route. During summer term there are three buses on the route. On July 21 we removed one at an arbitrary instant and observed the subsequent headways.

The graph of Figure 3 shows the results. The horizontal axis counts arrivals and the vertical axis the headways at the ISyE/Rec Center control point. According to the schedule, a complete loop of the route should take 36 minutes. With three buses, the observed headway was about 10 minutes (significantly less than the scheduled 36/3 = 12 minutes). We removed one of the three buses just after arrival #8, which left a large gap of just over 12 + 12 = 24 minutes. Under a schedule such a large headway would be expected to grow; but under our scheme, as predicted by our model, the headways of successive arrivals reéqualized at about 17 minutes (slightly less than the expected 18 minutes).
5 Comparative performance on a simulated route

We built a simulation of Route 63 of the Chicago Transit Authority (CTA). This route travels out and back along 63rd street between Stony Island Avenue and Midway Airport. The entire loop is 17.75 miles long (28.57 kilometers). We based our simulation on CTA data collected from GPS systems and automatic passenger counters on each bus.

Route 63 has almost 80 stops, of which the CTA monitors GPS data from only 18, including the two control points, one each at the easternmost and westernmost ends of the route. The historical travel times between key stops is well-described as the sum of uniformly distributed times for each intervening city block (1/8 mile or 0.2 kilometers in length).

We matched the simulated passenger arrivals and departures with the historical daily patterns by proceeding as follows: From the data we set the total arrivals to the system every half hour over a 14 hour period, from 04:00 to 18:00. Arrivals and departures at key bus stops vary over the day according to four major time periods: AM Early, AM Peak, Midday, and Evening Peak (Figure 4). The mean arrival rate for each particular bus stop
Figure 4: Arrival rates of passengers to CTA Route 63, showing morning and evening surges during a given time period was estimated by sampling from an exponential distribution with mean set to the mean number of boardings observed during that period. Dwell times at bus stops were computed based on the model of Milkovits (2008).

Figure 5 shows how the standard deviation of headways changed over a moderately busy 14-hour day under each of three control schemes: the target schedule used by the CTA, the target headway of Daganzo (2009), and the self-equalizing headways of Equation (3.1).

Under the target headway scheme of Daganzo (2009), a bus arriving at the control point is delayed for duration

\[ \max \{0, d + g (H - h_1)\}, \]  

(5.1)

where \( H \) is the target headway, \( h_1 \) is the forward headway of the arriving bus, \( d > 0 \) is the average delay at equilibrium, and \( g > 0 \) is a control parameter.

We ran many such simulations and these results are typical. Until about 08:00, all three schemes performed equally well, after which buses began to bunch under the target control.
Part of the reason that neither target schedules nor target headways worked well is that neither sufficiently accounts for increased load/unload times in a busy system, such as at rush hour, when dwell times can be super-linear. Apparently, self-equalizing headways had greater capacity to resist the consequent tendency to bunch.

6 Implementation

Our scheme is easy to implement. In the first experiment students computed the departure times for each successive bus. For the second experiment we provided the same functionality in wifi-enabled netbook computers. We are now building the third generation of the control system on mobile phones to be mounted in each bus. A subsequent paper will
describe details.

Because of its simplicity it is easy to adapt our scheme to account for additional business rules. For example, some bus systems guarantee drivers a short break time, \( \gamma \), at each arrival to specified control points. In this case, our scheme can be extended to delay each bus for either \( \gamma + \alpha h_n \) time units or until the previous bus has been gone for \( \beta \) time units, whichever is later. Self-equalization continues to assert itself: The argument of Theorem 3.1 holds, but with the time \( 1/\bar{v} \) to drive the route extended by the required break time \( \gamma \).

6.1 Choice of slack \( \alpha \)

By Expression 3.3, the value of \( \alpha \) is the slack at equilibrium and it can be set by the manager. Others have studied how to choose slack. For example, [Zhao et al. (2006)] used a queuing model to minimize passengers’ expected waiting times under schedule-based control. Under our scheme service is unscheduled and so the issues are somewhat different. In particular, choice of \( \alpha \) affects the speed of convergence and so the force with which self-equalization asserts itself. Analysis for small \( n \) and computational experiments for larger \( n \) suggest that convergence is fastest for values of \( \alpha \) in the range 0.5–0.6.

In any case, such a Markov Chain converges exponentially fast and therefore the time for our system to recover equal headways after a disturbance grows only logarithmically in the size of the disturbance.

6.2 Multiple control points

We analyzed self-equalizing headways from the point of view of a single control point, but one might prefer to have additional control points in longer routes, so that the spacing of the buses can be corrected more frequently. Because each control point makes successive headways more nearly equal, Theorem 3.1 can be generalized to show that using \( \alpha_j \) at
control point \( j = 1, \ldots, k \) propels convergence to a common headway of

\[
h^* = \frac{1}{\left(n - \sum_{j=1}^{k} \alpha_j\right) \bar{v}}. \tag{6.1}
\]

For example, there are two control points on the Georgia Tech route and the standard time allocated in the target schedule to drive the route is 29 minutes. To seek the same six minute headway of the schedule for six buses, we wrote Equation 6.1 as

\[
6 = \frac{29}{n - (\alpha_1 + \alpha_2)},
\]

from which it follows that \( \alpha_1 + \alpha_2 = 7/6 \). We could have chosen any solution for which \( 0 \leq \alpha_i \leq 1 \), but for convenience in doing hand calculations chose \( \alpha_1 = \alpha_2 = 7/12 \approx 0.5 \).

Under self-equalizing headways, the decisions at each control point are independent from those at other control points, so control points can be added or removed without interrupting operations.

7 Conclusions

7.1 Comparison with other methods of headway control

A target schedule sets static goals, and scheduled buses have little capacity to recover from disruptions. The only remedial action practical under a schedule is to delay a bus that is ahead of schedule, and the effect of this is only local. And a target schedule has no ability to respond to large disruptions. For example, if a bus breaks down, there remains a hole in the schedule, which tends to grow, until the bus can be replaced.

The target headway scheme of [Daganzo (2009)] is vulnerable to large disruptions. This was recognized by [Daganzo and Pilachowski (2010)], which also suggested another control scheme expected to better handle large disruptions. In this scheme, each bus relies on GPS to monitor the positions of the buses immediately ahead and behind. Then each bus speeds up or slows down to meet a pre-specified target headway.
This scheme seems promising. It is decentralized like ours and so shares some of the flexibility described in the next section. But it has greater communication requirements than ours, and it expects drivers to adjust their speeds, possibly frequently, which seems practical only in special circumstances such as reserved bus lanes. Furthermore, it remains to be seen to what extent one could actually exercise control over the bus drivers on the road. In contrast, our scheme leaves drivers alone to flow with the traffic.

Target headway schemes, such as those of Daganzo (2009) and Daganzo and Pila-chowski (2010), pose a target that is static and somewhat arbitrary. Road conditions and ridership levels change constantly and the achievable headway cannot be known a priori. Thus, a target headway is either too small and so merely aspirational, or else too large, and wasteful of bus capacity. In contrast, our scheme simply allows the achievable common headway to reveal itself.

7.2 Advantages of our scheme

Our scheme reduces work for management, and simplifies the job of the drivers, who can focus purely on flowing with the traffic. And most importantly, it provides better service to riders.

Under our control scheme the system of buses acts in effect as an analog computer and, in the absence of perturbations, computes the common headway by achieving it. A manager can adjust the resultant headway by changing the control parameter $\alpha$ or by inserting an additional bus into the flow or by removing one, and the headways will re-equilibrate appropriately.

It is unnecessary to construct a schedule in advance or to monitor adherence to a schedule. All that is required is the setting of two parameters, $\alpha$ to determine speed of convergence and bus utilization; and $\beta$, a lower bound on headway to speed equalization. The amount $\alpha$ of bus capacity held in reserve, is easy to understand and to control.

Our scheme adjusts the headway of each bus as it reaches a control point. It delays
the bus so that the new headway is a convex combination of its previous headway and the headway of the following bus (Expression 3.4). Large headways become smaller and small headways become larger, as they converge to a common value, which may be unknown or even changing.

Our scheme responds even to large disruptions, such as a bus breakdown or surge in ridership, by re-distributing the buses to equalize the headways at a new (larger) value. Similarly, with reduced traffic, the common headway is spontaneously reduced and service improves.

Another advantage of our scheme is that buses can be added to or removed from a route at arbitrary times and points. This is not so for a target schedule: For example, on the Georgia Tech Bus Route, scheduled service starts early in the day with a single bus, then increases to three, and finally to six; and in the evening service is reduced from six to three to one. It is impractical to increase the number of buses from three to, say, four, because the schedules of at least two of the three buses would have to be re-anchored, or else there be one headway twice the size of the others. In contrast, under our scheme, buses can be added or removed arbitrarily (though one may wish to adjust $\alpha$ and $\beta$).

Our scheme is robust even when the route is reconfigured, such as when construction altered the flow of traffic on campus and forced the temporary re-routing of the Georgia Tech buses. Such changes require construction of a new target schedule; but our scheme would continue to produce regular bus service.

Finally, under our scheme any improvements in processes, such as introducing procedures to speed boarding, go directly to reduce headways, without the need to rebuild schedules or re-compute target headways.

Self-equalizing headways seems especially useful when a transit system sets up a temporary service for a large public event, such as a state fair or sporting event. In such cases there may not be enough history of traffic velocities or ridership to produce a believable schedule.
For all these reasons, plus the ease of implementation, we think our system will be widely useful. Most immediately, we intend to try it for airport shuttles, such as for parking or rental cars. It may also be useful for trains.

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